

# LED intensity distribution

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**Abstract:** A radiometric approach to realistically model the intensity spatial distribution of encapsulated LEDs (light-emitting diodes) is presented. We provide an analytical relationship between the radiated pattern and the main LED parameters (chip, encapsulant, and reflector).

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**OCIS Codes:** (230.3670) Optical devices: light emitting diodes; (350.4600) Optical Engineering.

## 1. Introduction

Light-emitting diodes (LEDs) are semiconductor devices that can produce infrared, visible, or ultraviolet radiation. Their remarkable properties make them ideal sources for many applications ranging from indicator lights, displays, and optical communication systems to solid-state lighting. Although LEDs are widely applied in several areas of science and technology, there is not a realistic radiometric model for the emitted radiation distribution. LEDs are small extended sources with extra optics added to the chip, resulting in a complex intensity distribution difficult to model. Currently, the optical design of LEDs is carried out by means of Monte Carlo ray tracing methods. These techniques randomly simulate from 1 to 10 million light rays and the output ray-density distribution serves as an indirect value for the radiant intensity. In addition to the time consumed by these techniques, the lack of an analytic expression for the output intensity and irradiance reduces the optimization process to a trial and error procedure.

## 2. Radiometric model

The most frequently used quantity for radiometric characterization of LEDs is radiant intensity. In order to obtain the output intensity, we first consider the relationship between the chip radiance  $L$  ( $\text{W m}^{-2} \text{sr}^{-1}$ ) and the output radiance of LED. The radiance change across the boundary between two homogeneous isotropic media with indices of refraction  $n_1$  and  $n_2$ , and an angle of incidence at the interface  $\alpha_1$ , is  $L_2 = (n_2/n_1)^2 \tau(\alpha_1) L_1$  [1]. Here  $\tau(\alpha_1)$  is the radiant transmittance at the interface.

Radiant intensity  $I = \partial\Phi/\partial\Omega$  ( $\text{W sr}^{-1}$ ) is defined as the radiant flux from a point source emitted per unit solid angle in a given direction. In terms of source radiance, the intensity emitted by an infinitesimal source of area  $dA_s$  can be expressed as  $I = (L dA_s) \cos\theta_s$ . Therefore, the total radiated intensity by an extended source is given by the integration of the contributions of each infinitesimal area of the source [2]:

$$I_{total} = \iint_{source} L \cos\theta_s dA_s \quad , \quad (1)$$

where  $\cos\theta_s dA_s$  is the projected area of each infinitesimal source element. The near-field radiant intensity can be difficult to measure and interpret. However, in the far-field case, the meaning of total radiated intensity given by Eq. (1) becomes obvious. Indeed, in its publication 127, the Commission Internationale de l'Eclairage (CIE) only established the far-field conditions A and B for measurement of LED intensity. For the near-field case, the computation of irradiance  $E = \partial\Phi/\partial A$  ( $\text{W m}^{-2}$ ) could be more appropriate.

For an LED source (see Fig. 1), the intensity produced at a point located at  $\mathbf{r}$ , is

$$I_{out}(\mathbf{r}) = \frac{n_{out}^2}{n_{in}^2} \iint_{\substack{encapsulant \\ surface}} \tau[\alpha_{in}(\mathbf{r}_s; \mathbf{r})] L(\mathbf{r}_s; \mathbf{r}) \cos\alpha_{out}(\mathbf{r}_s; \mathbf{r}) dA \quad , \quad (2)$$

where the transmittance at the encapsulant dome is

$$\tau[\alpha_{in}(\mathbf{r}_s; \mathbf{r})] = \frac{n_{out} \cos[\alpha_{out}(\mathbf{r}_s; \mathbf{r})]}{n_{in} \cos[\alpha_{in}(\mathbf{r}_s; \mathbf{r})]} t[\alpha_{in}(\mathbf{r}_s; \mathbf{r})] \quad . \quad (3)$$

Here  $t$  is the Fresnel transmission coefficient, and  $\mathbf{r}_s$  is the position vector of an area element of the LED chip. Eq.(2) must be integrated over the output interface of the LED, i.e. the encapsulant. Trying to solve this integral for some easy geometries, we arrived at complex transcendental equations for the integration limits, thereby making the computation intractable. However, the radiance is conserved along any surface enclosed by the bunch of output rays, even the rays traced back [1]. To solve this problem we carry out the integral over the chip's paraxial image

formed by the encapsulation. This approach results in a considerable simplification limited only by small variations on the image contour due to distortion aberrations.

In this case, the LED intensity produced at a point located at  $\mathbf{r}$ , is

$$I_{out}(\mathbf{r}) = \frac{n_{out}^2}{n_{in}^2} \iint_{\substack{\text{chip} \\ \text{image}}} \tau[\alpha_{in}(\mathbf{r}_i; \mathbf{r})] L(\mathbf{r}_i; \mathbf{r}) \cos \theta_i(\mathbf{r}_i; \mathbf{r}) dA_i, \quad (4)$$

where  $\mathbf{r}_i$  is the position vector of the final image of an infinitesimal element of the LED chip.

Assuming that the emitted light is not polarized, and write the transmittance as

$$\tau[\alpha_{in}(\mathbf{r}_i; \mathbf{r})] = \frac{n_{out} \cos \alpha_{out}(\mathbf{r}_i; \mathbf{r})}{2n_{in} \cos \alpha_{in}(\mathbf{r}_i; \mathbf{r})} (t_{\perp}^2[\alpha_{in}(\mathbf{r}_i; \mathbf{r})] + t_{\parallel}^2[\alpha_{in}(\mathbf{r}_i; \mathbf{r})]), \quad (5)$$

where  $t_{\parallel}$  and  $t_{\perp}$  are the Fresnel transmission coefficients at the encapsulant surface for the electric fields parallel and perpendicular to the plane of incidence, respectively. An appropriate representation of Eq. (5) for faster integral computation is:

$$\tau[\alpha_{in}(\mathbf{r}_i; \mathbf{r})] = \frac{\sin[2\alpha_{in}(\mathbf{r}_i; \mathbf{r})] \sin[2\alpha_{out}(\mathbf{r}_i; \mathbf{r})]}{2 \sin^2[\alpha_{in}(\mathbf{r}_i; \mathbf{r}) + \alpha_{out}(\mathbf{r}_i; \mathbf{r})]} \{1 + \sec^2[\alpha_{in}(\mathbf{r}_i; \mathbf{r}) - \alpha_{out}(\mathbf{r}_i; \mathbf{r})]\}. \quad (6)$$

The angle of incidence  $\alpha_{in}$  is computed by Snell's law from  $\alpha_{out}$ , and the outgoing angle  $\alpha_{out}$  is geometrically given by:

$$\alpha_{out}(\mathbf{r}_i; \mathbf{r}) = \arcsin \left[ \frac{|\mathbf{r}_2(\mathbf{r}_i; \mathbf{r}) \times \mathbf{S}(\mathbf{r}_i; \mathbf{r})|}{|\mathbf{r}_2(\mathbf{r}_i; \mathbf{r})| |\mathbf{S}(\mathbf{r}_i; \mathbf{r})|} \right], \quad (7)$$

where  $\mathbf{r}_2 = \mathbf{r} - \mathbf{r}_i$ , and  $\mathbf{S}$  is the surface normal vector of the dome.

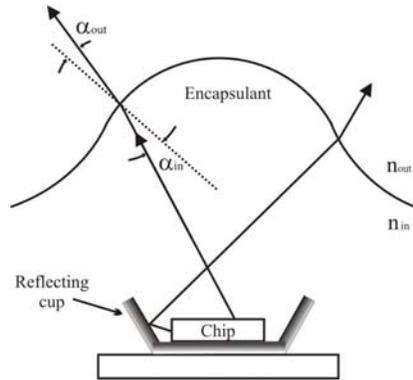


Fig. 1. Propagation of radiation inside an LED.

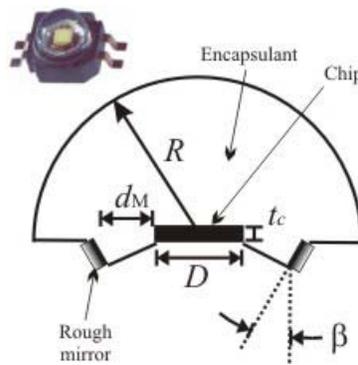


Fig. 2. Simplified schematic diagram of model for the Luxeon® K2 LED.

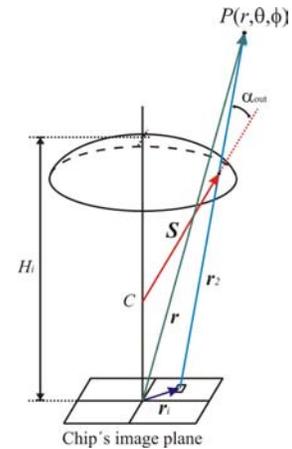


Fig. 3. Illustrating notation relating to formula (9).

### 3. Example: LUXEON® K2 LED

As an interesting example and proof of our approach, we analyze the latest high-power LED of Lumileds®, named LUXEON® K2. This LED has a spherical encapsulant, and a flat square chip located at the center of the dome with a ratio  $R/D \sim 1.7$  (where  $R$  is the encapsulant curvature radius and  $D$  is the chip lateral size, see Fig. 2). Additionally, the chip is surrounded by six rough mirrors (inset in Fig. 2). Radiance for each face of the chip is assumed to be Lambertian,  $L=L_0$ .

First we consider the LUXEON® K2 white emitter. This is a nearly perfect Lambertian source (probably due to the phosphor layer that avoids the pattern shapes caused by the lateral mirrors). Therefore, we consider only the upper flat chip surface, i.e., a plane square source. The far-field intensity, produced at a point located at  $\mathbf{r}=(r, \theta, \phi)$ , becomes

$$I_{out}(\theta, \phi) = \frac{n_{out}^2}{n_{in}^2} L_0 \cos \theta \int_{-D/2}^{D/2} \int_{-D/2}^{D/2} \tau[\alpha_{in}(x_i, y_i; \theta, \phi)] dx_i dy_i, \quad (8)$$

where

$$\alpha_{out}(x_i, y_i; \theta, \phi) = \arcsin \left\{ R^{-1} \left[ (y_i \cos \theta + H_i \sin \phi \sin \theta)^2 + (x_i \cos \theta + H_i \cos \phi \sin \theta)^2 + (y_i \cos \phi \sin \theta - x_i \sin \phi \sin \theta)^2 \right]^{\frac{1}{2}} \right\}. \quad (9)$$

Here  $H_i$  is the distance from the chip image position to the dome apex (see Fig. 3), and  $D_i = mD$  (where  $m$  is the transverse magnification). For simplicity we locate the coordinate system at the center of the image plane.

Fig. 4 shows the radiation patterns for  $n_{out}=1$ ,  $n_{in}=1.5$ ,  $\phi=0$ , and some selected values of the  $H/R$  ratio. If the LUXEON® K2 LED chip is centered in the hemi-spherically shaped encapsulant, then  $H/R=1$ , and the computed intensity perfectly agrees with the datasheet for white emitter [3]. The patterns for the other values of the  $H/R$  agree well with the intensity distributions (for more directional LEDs) reported by many other manufacturers (e.g. Nichia® LEDs).

For green, cyan, blue and royal blue LUXEON® K2 LEDs, the intensity pattern is some what more complex. To model these LEDs, we consider the contributions of the upper and the lateral surfaces of the chip, and the reflection at the rough mirrors. We approximate the radiance reflected by the mirrors by the scattering angle approach

$$L(\mathbf{r}_i; \theta, \phi) = \gamma L_0 \cos^N [\psi(\mathbf{r}_i; \theta, \phi)] = \gamma L_0 \left[ \frac{\mathbf{r}_2(\mathbf{r}_i; \mathbf{r}) \cdot \mathbf{r}_{specular}(\mathbf{r}_i)}{|\mathbf{r}_2(\mathbf{r}_i; \mathbf{r})| |\mathbf{r}_{specular}(\mathbf{r}_i)|} \right]^N, \quad (10)$$

where  $\mathbf{r}_{specular}$  is the direction vector of the specular reflection at each point  $\mathbf{r}_i$  over the mirror surface, and  $N$  is the scattering factor associated with the roughness of the mirror. Fig. 5 shows the radiation patterns for  $n_{out}=1$ ,  $n_{in}=1.5$ ,  $\phi=0$ ,  $R/D=1.7$ ,  $d_M/D=0.8$ ,  $t_c=D/10$ ,  $\gamma=1$ ,  $N=50$ , and some selected values of the angle  $\beta$ . The computed intensity agrees well with the datasheet for the green, cyan, blue and royal blue emitters [3], in particular for the value of  $\beta=70^\circ$ .

#### 4. Conclusion

We have presented what is to our knowledge the first radiometric approach to realistically model the intensity spatial distribution of LEDs. The model provides an analytical relationship between the radiated pattern and the main parameters of LEDs, which include: chip shape, chip radiance, encapsulant geometry, encapsulant refractive index, chip location, and cup reflector. The validity of the model was demonstrated by the excellent agreement between our results and experimental data sheets of several manufacturers. This relationship could be a powerful analytic tool in the optical design of LEDs over a wide range of LED parameters. For example, our model could be applied to the determination of the optical tolerances upon an LED, which could be one of the most important subjects in optical design of LEDs. Another possible application is the exact design of LED arrays, which is currently done using first order approximations for the intensity distribution [4-5].

This research was supported by CONACYT (Consejo Nacional de Ciencia y Tecnologia) grant J48199-F.

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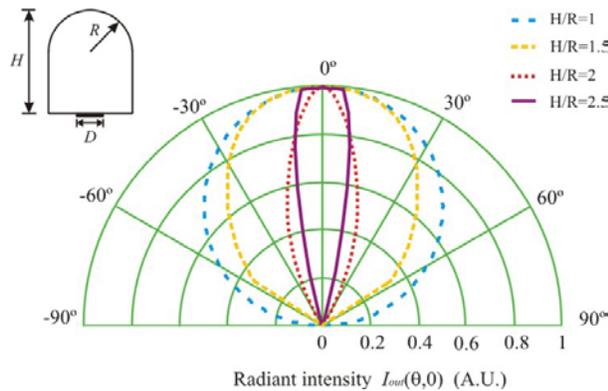


Fig. 4. Far-field intensity distribution (normalized to its maximum value) computed using the proposed radiometric model for  $R/D=1.7$ . Note that the pattern  $H/R=1$  corresponds to a perfect Lambertian emitter with  $\theta_{1/2}=60^\circ$ . The other angles  $\theta_{1/2}$  with width at half-maximum intensity are:  $46.6^\circ$  ( $H/R=1.5$ ),  $17^\circ$  ( $H/R=2$ ), and  $11^\circ$  ( $H/R=2.5$ ).

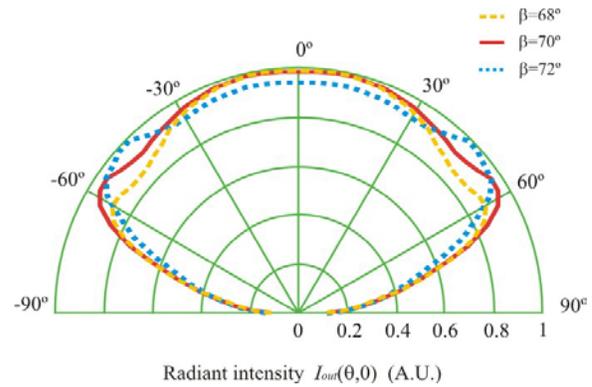


Fig. 5. Far-field intensity distribution for the green, cyan, blue and royal blue LUXEON® K2 LEDs.