# Thin-film spatial filters

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A thin-film optical filter used as a one-dimensional spatial filter is presented, and its design is briefly examined. The filter consists of a stack of quarter-wave dielectric layers upon a right-angle prism that selectively cancel a reflected or transmitted plane-wave front for various angles of incidence. Transmittance and reflectance are low-pass functions or high-pass functions of the angle of incidence with a high degree of steepness. In combination, these filters exhibit bandpass transmittance with a variable bandwidth. Applications to detection of extrasolar planets are briefly discussed. © 2005 Optical Society of America

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Spatial filtering has been broadly applied to image enhancement and image processing in several regions of the electromagnetic spectrum. Currently, modern spatial-frequency filtering is performed by use of interference patterns,[1] anisotropic media,[2] liquid-crystal cells,[3] or resonator grating systems.[4] Spatial filters for optical applications generally are two dimensional. However, there are some applications that require direct one-dimensional filtering with a high degree of steepness. One of the most appropriate and useful applications is found in detection of extrasolar planets. For detection of a faint planet in the vicinity of a bright star, the planet's plane-wave spectrum must be isolated from the star's plane-wave spectrum and also must be detected.[5,6]

Our purpose in this Letter is to examine briefly the use of optical thin films as one-dimensional spatial filters. Because a wave front can be decomposed into a distribution of plane waves with a range of inclination angles, we convert the transmittance and reflectance into edge functions of the incidence angle. The device is simple; it consists, in its entirety, of a stack of quarter-wave dielectric layers deposited upon a right-angle prism (Fig. 1).

Spectral filters remain the most popular application of optical thin films. The main application of thin films for oblique incidence is in beam splitters. The reflectance and transmittance at oblique incidence show a strong dependence on the angle of incidence when the films are within prisms.[7] Designs of beam-splitter cubes are then optimized to accommodate both a large spectral range and a large angular field of view.[8] This strong dependence on the incidence angle can be used to design a thin-film filter to cancel a reflected plane-wave front that is incident at an angle slightly smaller than that of the total internal reflection; this causes the filter's reflectance to change greatly when the angle of incidence varies incrementally. Thus transmittance and reflectance become edge functions of the angle of incidence and, therefore, of the spectrum of spatial frequencies.

The performance of thin films at oblique angles of incidence is equivalent to the normal-incidence performance of two designs, one for each polarization. For $p$ polarization, each index of refraction in the design, including the massive media indices, is replaced by the effective index $n_p = n/\cos\varphi$; for $s$ polarization, $n_s = n\cos\varphi$.[9] Here $n$ is the index of refraction and $\varphi$ is the angle of incidence in that layer. With the use of Snell's law, $\cos\varphi = (1 - S^2/n^2)^{1/2}$, where $S = n_M \sin\theta_M$ and $\theta_M$ and $n_M$ are the angle of incidence and the index of refraction in the entrance medium, respectively. In a film system, $S$ is a constant in all layers, so $n_p$ and $n_s$ may be written as $n_p = n^2(n^2 - n_M{}^2 \sin^2\theta_M)^{-1/2}$ and $n_s = (n^2 - n_M{}^2 \sin^2\theta_M)^{1/2}$. The reflectance of a dielectric thin-film system is given by[9] $R = [(n_M - Y)/(n_M + Y)]^2$, where $Y$ is the admittance of the system. At oblique angles of incidence, one obtains the reflectance values of the $p$ polarization and the $s$ polarization by replacing all the indices with effective indices $n_p$ and $n_s$, and the optical thicknesses are multiplied by $(\cos\varphi)^{-1}$.

Antireflection can be obtained from a stack of quarter-wave dielectric layers of alternate high (H) and low (L) index. In this case, when high-index layers are arranged outermost from both sides of the stack, the admittance is given by[9]

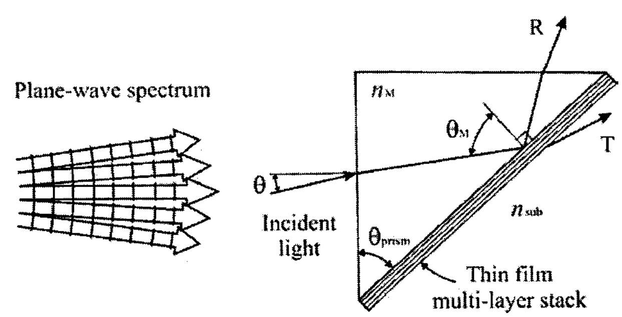


Fig. 1. Thin-film filter for spatial frequencies assembled with a multilayer stack upon a glass prism; $\theta_{\mathrm{prism}}$ is the angle of the prism, $n_M$ is the index of refraction of the incident medium, and $n_{\mathrm{sub}}$ is the index of the substrate medium.

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$$Y = \left(\frac{n_H}{n_L}\right)^{2m} \frac{n_H{}^2}{n_{\text{sub}}}; \tag{1}$$

when low-index layers are arranged outermost from both sides of the stack, the admittance is

$$Y = \left(\frac{n_L}{n_H}\right)^{2m} \frac{n_L^2}{n_{\text{sub}}}, \tag{2}$$

where $n_{\text{sub}}$, $n_H$, and $n_L$ are the indices of the substrate, the high-index layers, and the low-index layers, respectively. The number of layers in the stack is $(2m + 1)$.

We can choose quarter-wave effective optical thicknesses (in each layer) and find film index values for antireflection, both at a design angle of incidence $\theta_M = \theta_0$. To produce zero reflection at angle of incidence $\theta_M = \theta_0$, the film index value $n_L$ for $s$ polarization is

$$n_L = \left\{ \left[ \frac{\cos\theta_0}{(n_M n_{\text{sub}})^m} \left(1 - \frac{n_M{}^2}{n_{\text{sub}}{}^2} \sin^2\theta_0\right)^{1/2} (n_H{}^2 - n_M{}^2 \sin^2\theta_0)^m \right]^{1/m+1} + \frac{n_M}{n_{\text{sub}}} \sin^2\theta_0 \right\}^{1/2} (n_M n_{\text{sub}})^{1/2}, \tag{3}$$

and for $p$ polarization the film index value $n_H$ is

$$n_H = \left( \frac{1 + \left\{ 1 - 4 n_M{}^2 \sin^2\theta_0 \left[ \frac{\cos\theta_0}{n_M n_{\text{sub}}} \left(1 - \frac{n_M{}^2}{n_{\text{sub}}{}^2} \sin^2\theta_0\right)^{1/2} \left( \frac{n_L{}^2 - n_M{}^2 \sin^2\theta_0}{n_L{}^4} \right)^m \right]^{1/m+1} \right\}^{1/2}}{2 \left[ (n_M n_{\text{sub}})^m \cos\theta_0 \left(1 - \frac{n_M{}^2}{n_{\text{sub}}{}^2} \sin^2\theta_0\right)^{1/2} \left( \frac{n_L{}^2 - n_M{}^2 \sin^2\theta_0}{n_L{}^4} \right)^m \right]^{1/m+1}} \right)^{1/2} (n_M n_{\text{sub}})^{1/2}. \tag{4}$$

Equation (3) is for a filter with low-index layers arranged outermost from both sides of the stack, and Eq. (4) corresponds to a filter with high-index layers arranged outermost from both sides of the stack.

Equations (3) and (4) give the design conditions for a single-layer spatial filter when $m=0$, and these expressions result in the well-known condition for normal incidence ($\theta_0 = 0°$), i.e., the square root of the product of $n_M$ and $n_{\text{sub}}$.

To create a spatial filter with a high degree of edge steepness as a function of angle of incidence $\theta_M$, the reflectance slope is maximized. The highest reflectance slope appears near the angle of total internal reflection (TIR), $\theta_C = \sin^{-1}(n_{\text{sub}}/n_M)$. This optical effect can be used to design a thin-film filter to cancel the reflected plane-wave front if it is incident at an angle $\theta_0$ slightly smaller ($\Delta\theta_C$) than that of the TIR at the film–substrate interface; we then choose

$$\theta_0 = \sin^{-1}(n_{\text{sub}}/n_M) - \Delta\theta_C \tag{5}$$

as a design expression. Then Eqs. (3)–(5) become the design expressions for a quarter-wave multilayer spatial filter.

Figure 2 shows the reflectance and transmittance of a multilayer filter (deposited over a right-angle prism) as a function of angle of incidence $\theta$ at the prism entrance (Fig. 1). These figures are for $n_M = 1.5$, $n_{\text{sub}} = 1$ (air), $\Delta\theta_C = 0.1°$, and $m=2$ (five layers); all the layers are one quarter-wave thick. For the $s$ polarization $n_H = 2.1$, $n_L = 1.389$, and the design is 1.5|L H L H L|air. For $p$ polarization $n_H = 2.615$, $n_L = 1.38$, and the design is 1.5|H L H L H|air. The reflectance and transmittance functions act as high-pass filters for angles of incidence in one direction and as low-pass filters in the opposite direction. The dotted curves incorporate Fresnel losses at the lateral faces of the prism. The degree of edge steepness of these reflectance and transmittance functions can be improved by the addition of more layers to the design. This is particularly important in some applications, such as detection of extrasolar planets, that require an exceptional degree of edge steepness.

Extensive investigations are being conducted to find Earth-like planets around other stars. Overcoming the high brightness contrast and the small angular separation between a star and its companion planet remains the most difficult challenge in this search. The optical filters that we have discussed would offer an entirely new method (by itself or as a complement) of direct detection of extrasolar planets. By means of TIR these filters can obtain the reflected planetary signal and produce zero reflected intensity for the star signal, which must be made to input at an angle slightly less than the angle of TIR. Typical star–planet angular separations at Earth are $\sim 2 \times 10^{-6}$ rad ($1.15 \times 10^{-4}$ deg). A simple quarter-wave multilayer design for $s$ polarization, 1.9|L H L H L H L|air, with $n_H = 2.4$, $n_L = 1.35$, $n_{\text{sub}} = 1$ (air), and $m=3$ (seven layers) permits a very

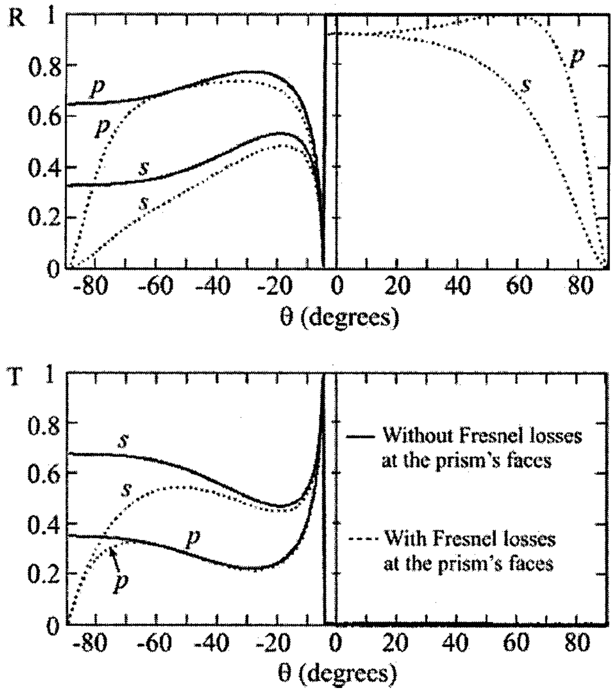


Fig. 2. Top, reflectance and bottom, transmittance of a multilayer design as a function of angle of incidence for $n_M=1.5$, $n_{\mathrm{sub}}=1$ (air), $\Delta\theta_C=0.1°$, $\theta_{\mathrm{prism}}=45°$ (right-angle prism), and $m=2$ (five layers); all the layers are one quarter-wave thick at $\theta=-4.938°$ ($\theta_0=41.71°$). For $s$ polarization, $n_H=2.1$, $n_L=1.389$, and the design is 1.5|L H L H L|air. For $p$ polarization, $n_L=1.38$, $n_H=2.615$, and the design is 1.5|H L H L H|air.

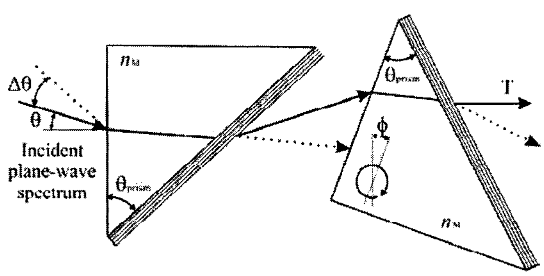


Fig. 3. Bandpass spatial filter assembled with two edge thin-film spatial filters like the filter shown in Fig. 1. The second prism is rotated by $\phi$ to control bandwidth $\Delta\theta$.

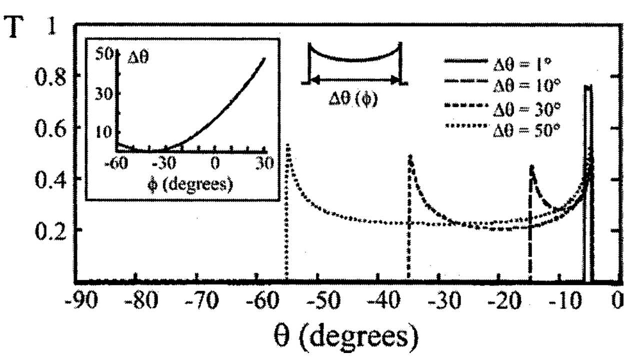


Fig. 4. Transmittance of the bandpass filter shown in Fig. 3. The design of the edge filters is that which was analyzed for $s$ polarization in Fig. 2. Bandwidth $\Delta\theta$ is variable with angle of prism rotation $\phi$.

small angular separation ($\Delta\theta_c=1.15\times10^{-4}$ deg) between TIR and interference suppression. Furthermore, one can improve this degree of edge steepness by adding more layers to the stack. These spatial filtering capabilities reveal the feasibility of detecting a signal generated by an extrasolar planet.

To design a bandpass filter, a suitable construction would be a combination of the previously analyzed high-pass and low-pass filters (Fig. 3). Figure 4 shows the transmittance as a function of the angle of incidence (all Fresnel losses are incorporated). We note that the bandpass filter displays a variable bandwidth $\Delta\theta$, tunable with the amount of prism rotation $\phi$. The bandwidth is given by

$$\Delta\theta(\phi)=\sin^{-1}\left(n_M\sin\left\{\theta_{\mathrm{prism}}-\sin^{-1}\left[\frac{1}{n_M}\sin(\psi_0+\theta_{\mathrm{prism}}-\phi)\right]\right\}\right)-\psi_0, \quad (6)$$

where $\phi_0=\sin^{-1}\{n_M\sin[\theta_{\mathrm{prism}}-\sin^{-1}(n_M{}^{-1})]\}$.

In summary, we have proposed a new application of optical thin films as filters for spatial frequencies and have briefly examined the design principles. The filters have a transmittance and reflectance that are edge functions of the angle of incidence. In combination, these filters showed a bandpass transmittance with variable bandwidth. We deduced the design equations for a quarter-wave stack of dielectric thin films over one right-angle prism for $s$ and $p$ polarization. Reflectance and transmittance as a function of the angle of incidence showed a higher degree of edge steepness when the layer number was increased. The high degree of steepness makes this spatial filter ideal for detection of extrasolar planets. Other possible applications include high-sensitivity angle sensors[10] and beam-splitter cubes for special purposes.[11]

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